Int J Theor Phys (2009) 48: 894–904
DOI 10.1007/s10773-008-9862-3

Entanglement in Bipartite Generalized Coherent States

S. Sivakumar

Received: 16 June 2008 / Accepted: 23 September 2008 / Published online: 3 October 2008

Abstract Entanglement of formation in a class of bipartite generalized coherent states is discussed. It is shown that a positive parameter can be associated with these bipartite states so that the states with equal value for the parameter are of equal entanglement. For the class of states considered, the maximum possible entanglement of one ebit is attained if the value of the positive parameter is $\sqrt{2}$. It is shown that the entanglement of formation is one ebit when the relative phase between the composing states is π in the class of bipartite generalized coherent states considered.

Keywords Generalized coherent states · Entanglement · Concurrence

1 Introduction

Quantum entanglement is essential for quantum communication. All the gadgets, such as the teleportation devices, quantum repeaters and entanglement distillers, use entanglement [3–5, 32] to make the quantum communication possible. The EPR paradox and the Bell's inequalities which clarify many of the fundamental issues of the quantum theory arise as consequences of the quantum entanglement [26]. Quantum computers also need the entanglement in fundamental ways for their operation [12, 23]. The entanglement properties of bipartite systems whose associated Hilbert spaces are of dimension two or three are well understood [15, 16, 27]. However, the entanglement of states defined on infinite dimensional Hilbert spaces possess many interesting features. For instance, entangled states are generic in a bipartite system, if one or both the subsystems have an infinite dimensional Hilbert space [8]. Further, the entanglement is unlimited in contrast to the two-level systems where it is limited to one ebit. States with such high entanglement are dense in the infinite dimensional cases [11]. Study of such states with arbitrarily large amount of entanglement is important to understand the decoherence mechanisms of qubits coupled with the environment [19]. Electromagnetic field is by far the most successful communication vehicle. The

S. Sivakumar (✉)
Materials Science Division, Indira Gandhi Centre for Atomic Research, Kalpakkam 603 102, India
e-mail: siva@igcar.gov.in

states of the quantized electromagnetic field belong to infinite dimensional Hilbert spaces. The possibility of optical realization of these states renders them suitable for quantum communication [6]. Recently, the relation between the entanglement of quantum states and the Lie algebraic structures has been established [2].

A well-studied example of entanglement in infinite dimensional states is the entangled, bipartite coherent state $|\alpha,\phi\rangle$ [28, 31], defined as

$$|\alpha,\phi\rangle = N_e[|\alpha\rangle|\alpha\rangle + \exp(i\phi)|-\alpha\rangle|-\alpha\rangle]. \tag{1}$$

Here $|\alpha\rangle$ is a single mode coherent state (CS) of complex amplitude α. The normalization constant N_e is $1/\sqrt{2[1+\cos\phi\exp(-4|\alpha|^2)]}$. The entangled CS of the electromagnetic field is realizable in the interaction of a CS with a 50:50 beam splitter in a Mach-Zander interferometer [28], in the cross-phase-modulation in the electromagnetically induced transparency [24] etc. Multidimensional bipartite coherent states are extensions of the two-mode CS. They involve more than two product states in the superposition and practical schemes are known to create them [22, 34].

An interesting result concerning the entangled CS $|\alpha,\phi\rangle$ is that they possess exactly one ebit of entanglement when the relative phase ϕ is π. Further, for this special value of the relative phase, the entanglement in $|\alpha,\phi\rangle$ is independent of the amplitude α [14]. These states have been studied in various contexts related to the teleportation [33], the entanglement purification [9], the universal quantum computing [17] and the Bell's inequalities [18]. For other values of ϕ, the entanglement in $|\alpha,\phi\rangle$ is one ebit if the amplitude α becomes very large. Though the composing states in (1) belong to infinite dimensional Hilbert spaces, the entangled CS itself belongs to a four dimensional subspace of the tensor product of the two spaces associated with the two modes. The two orthogonal states $|\pm\rangle = n_\pm[|\alpha\rangle \pm |-\alpha\rangle]$, provide a two-dimensional basis to express the states $|\alpha\rangle$ and $|-\alpha\rangle$. The normalisation coefficients are $n_\pm = 1/\sqrt{2[1\pm\exp(-2|\alpha|^2)]}$. To express the entangled CS defined in (1), two-mode extensions of $|\pm\rangle$ are required. There are four such states, namely, $|+,+\rangle$, $|+,-\rangle$, $|-,+\rangle$ and $|-,-\rangle$. These four states are normalized as they are product states composed out of the normalized states $|\pm\rangle$. Following the work of [4] for the qubits, the *magic basis* [13] suitable for the entangled CS is

$$|E_1\rangle = \frac{1}{\sqrt{2}}\left[|+,+\rangle + |-,-\rangle\right], \tag{2}$$

$$|E_2\rangle = \frac{i}{\sqrt{2}}\left[|+,+\rangle - |-,-\rangle\right], \tag{3}$$

$$|E_3\rangle = \frac{i}{\sqrt{2}}\left[|+,-\rangle + |-,+\rangle\right], \tag{4}$$

$$|E_4\rangle = \frac{1}{\sqrt{2}}\left[|+,-\rangle - |-,+\rangle\right]. \tag{5}$$

The superpositions of the type given in (1) are easily mapped on to the Bell type superpositions of spin-1/2 particles since the states $|\alpha\rangle$ and $|-\alpha\rangle$ can be expressed as a linear combination of the states $|+\rangle$ and $|-\rangle$. These two states play the role of the spin-up and the spin-down states of a spin-1/2 particle. A nice feature of the magic basis is that their linear superpositions are maximally entangled whenever the superposition coefficients are real. It is important to note that this magic basis is not useful for other possible two-mode coherent state superpositions. For instance, the states of the form $|\alpha\rangle|\gamma\rangle + \exp(i\phi)|\gamma\rangle|\alpha\rangle$, where $|\gamma\rangle$

and $|\alpha\rangle$ are coherent states of amplitudes of different magnitudes, are not expressible as linear superpositions of the magic basis. The given magic basis states are suitable only for those superpositions which involve the two-mode extensions of the coherent states of amplitudes α and $-\alpha$. Once the magic basis is available, the results of [5] on the two-state entangled systems are readily extended to the entangled CS. For the sake of completeness, their result on the bipartite systems is presented here: if a bipartite pure state is expressed in the magic basis as $\sum_{j=1}^{4} c_j|E_j\rangle$, the *concurrence* C between the subsystems is $|\sum_{j=1}^{4} c_j^2| \leq 1$. Defining $x = .5 + .5\sqrt{1-C^2}$, the entanglement of formation E associated with the bipartite system is

$$E = -\left[x \log_2 x + (1-x)\log_2(1-x)\right]. \tag{6}$$

The quantities C are E are related in a special way, namely, an increase or a decrease of the one implies the same behaviour for the other. Both C and E vanish for any separable state and have nonzero values for any entangled state. Other well known measures of the bipartite entanglement are the von Neumann entropy $S = \mathrm{Tr}[\hat{\rho}_c \log \hat{\rho}_c]$ and the linear entropy $L = 1 - \mathrm{Tr}[\hat{\rho}_c^2]$. Here $\hat{\rho}_c$ is the reduced density operator for any one of the subsystems and Tr is used to denote the trace of an operator. For a pure state, the concurrence C and the linear entropy L are related through $C = \sqrt{L}$. As far as the pure states are concerned, these measures are equivalent. The quantum states studied in this paper are bipartite, pure states. Since the various measures of entanglement are equivalent for the pure states, any one of them can be used to quantify the entanglement. In this work, the concurrence C is used as the measure of entanglement.

The entangled CS defined in (1) is expressed in the magic basis as

$$\begin{aligned}|\alpha,\phi\rangle = \frac{N_e}{\sqrt{2}}\big[(1+\exp(i\phi))\big[|E_1\rangle - i\exp(-2|\alpha|^2)|E_2\rangle\big] \\ - i\sqrt{1-\exp(-4|\alpha|^2)}(\exp(i\phi)-1)|E_3\rangle\big].\end{aligned} \tag{7}$$

When $\phi = \pi$, the state $|\alpha,\phi\rangle$ becomes the magic basis state $|E_3\rangle$, apart from a phase factor. Hence, the state $|\alpha,\pi\rangle$ has one ebit of entanglement [14]. While the CS states are important for their interesting physical and mathematical properties [20], a variety of other states of the electromagnetic field are generated in the interaction of the electromagnetic fields with nonlinear media. These states may posses properties such as squeezing, anti-bunching, sub-Poissonian statistics, etc. The properties are entirely quantal in origin [21, 29]. Mathematically, these states are obtained either as the eigenstates of a suitable annihilation operator or by the action of an unitary operator on the ground state of the electromagnetic field. These states are referred as Generalized Coherent States (GCS) [25, 35]. The usual CS $|\alpha\rangle$ form a subset of the GCS, obtained as the eigenstates of the harmonic oscillator annihilation operator $\hat{a}$. Equivalently, the CS are obtained by the action of the unitary operator $\exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a})$ on the ground state $|0\rangle$. A GCS $|\beta\rangle$ of the electromagnetic field expanded in the number states basis is

$$|\beta\rangle = \sum_{n=0}^{\infty} C_{kn}\beta^{kn}|nk+m\rangle, \quad \beta \in C. \tag{8}$$

The coefficients satisfy $\sum_{n=0}^{\infty}|C_{kn}\beta^{kn}|^2 = 1$, the usual normalization condition. The magnitude of β is restricted by this normalization requirement. The coefficients C_{kn} are functions of n and $|\beta|$. The parameter k is a positive integer and m is a non-negative integer.

Table 1 k, m and c_n for various coherent states

State	k	m	C_{kn}
CS	1	0	$\frac{\exp(-\frac{\lvert\beta\rvert^2}{2})}{\sqrt{n!}}$
SV	2	0	$\frac{\sqrt{(kn)!}}{n!\sqrt{\cosh(\lvert\beta\rvert^2)}}[\frac{\tanh(\lvert\beta\rvert^2)}{2\lvert\beta\rvert^2}]^n$
ECS	2	0	$\frac{1}{\sqrt{\cosh(\lvert\beta\rvert^2)(kn)!}}$
OCS	2	1	$\frac{\lvert\beta\rvert}{\sqrt{\sinh(\lvert\beta\rvert^2)(kn+1)!}}$
LS	1	0	$\frac{\gamma}{\sqrt{\lvert\gamma\rvert^2-\log(1-\lvert q\rvert^2)}}$ for $n=0$
			$\frac{1}{n\sqrt{\lvert\gamma\rvert^2-\log(1-\lvert q\rvert^2)}}$ for $n>0$

The relevant values of the parameters for the CS and some of the well known GCS such as the squeezed vacuum (SV) [7], the even coherent states (ECS), the odd coherent states (OCS) [10] and the logarithmic states (LS) [30] are given in Table 1. The states $|+\rangle$ and $|-\rangle$ introduced earlier define the ECS and the OCS respectively. The dependence of C_{kn} on n and $|\beta|$ are indicated in the last column of Table 1. In the case of the logarithmic states, the amplitude q is real and its magnitude is less than unity. The parameter γ is complex and unrestricted.While the discussion is restricted to the GCS in this work, it has been established that the concurrence is a measure of entanglement for the parasupersymmetric CS as well [1].

In this work, it is established that all the entangled GCS, defined in (18), attain one ebit of entanglement whenever the relative phase ϕ is π. This is an extension of the corresponding result for the usual CS. Further, the condition to attain one ebit of entanglement irrespective of the relative phase is given. Analytical results are presented for the states listed in Table 1.

2 Entanglement in Two-Mode Generalized Coherent States

The ECS and the OCS provide a two-dimensional basis to express the states $|\alpha\rangle$. Similarly, the following orthogonal states are defined for the GCS $|\beta\rangle$:

$$|\beta+\rangle = N_+[|\beta\rangle + |(-1)^{\frac{1}{k}}\beta\rangle] = A\sum_{n=0}^{\infty} C_{2nk}\beta^{2n}|2nk+m\rangle, \tag{9}$$

$$|\beta-\rangle = N_-[|\beta\rangle - |(-1)^{\frac{1}{k}}\beta\rangle] = B\sum_{n=0}^{\infty} C_{(2n+1)k}\beta^{2n+1}|(2n+1)k+m\rangle. \tag{10}$$

It is to be noted that the states $|\beta\rangle$ and $|(-1)^{\frac{1}{k}}\beta\rangle$ are used in constructing the orthogonal states. The constants N_+ and N_- are for the normalization of the states. The parameters A and B are positive. In terms of the expansion coefficients C_{nk}, these parameters are given by the expressions,

$$A = \left[\sum_{n=0}^{\infty} |C_{2nk}\beta^{2nk}|^2\right]^{-\frac{1}{2}}, \tag{11}$$

$$B=\left[\sum_{n=0}^{\infty}|C_{(2n+1)k}\beta^{(2n+1)k}|^2\right]^{-\frac{1}{2}}. \tag{12}$$

The parameters satisfy $A^{-2}+B^{-2}=1$ as a consequence of the normalization of the states. The $|\beta|$-dependence of the A parameters for the states listed in Table 1 are:

$$A_{CS}=\sqrt{\frac{2}{1+\exp(-2|\beta|^2)}}, \tag{13}$$

$$A_{SV}=\sqrt{\frac{2}{1+\sqrt{1-\tanh(2|\beta|^2)\tanh(|\beta|^2)}}}, \tag{14}$$

$$A_{ECS}=\sqrt{\frac{2\cosh(|\beta|^2)}{\cos(|\beta|^2)+\cosh(|\beta|^2)}}, \tag{15}$$

$$A_{OCS}=\sqrt{\frac{2\sinh(|\beta|^2)}{\sin(|\beta|^2)+\sinh(|\beta|^2)}}, \tag{16}$$

$$A_{LS}=\sqrt{\frac{|\gamma|^2-\log(1-|q|^2)}{|\gamma|^2-\frac{1}{2}\log(1-|q|^4)}}. \tag{17}$$

The suffixes indicate the relevant states. The asymptotic value of A is $\sqrt{2}$ for the states listed in Table 1. This limit is obtained when $|\beta|\to\infty$. For the LS the corresponding limit is obtained when $|q|\to 1$. In Fig. 1 the dependence of A on the magnitude of the complex amplitude β is shown for the various states given in Table 1. The LS are specified by two parameters, namely, the complex γ and the real q. So the function A_{LS} is not shown in Fig. 1. It is to be noted that the A parameter of the ECS and the OCS are oscillatory in $|\beta|$. The peak values of these oscillations are larger than the asymptotic value.

The GCS $|\beta\rangle$ and $|(-1)^{\frac{1}{k}}\beta\rangle$ can be written linear combinations of the states $|\beta+\rangle$ and $|\beta-\rangle$. As in the case of the CS, the basis provided by $|\beta+\rangle$ and $|\beta-\rangle$ is suitable to expand a GCS whose amplitude is either β or $(-1)^{1/k}\beta$. This two-dimensional basis facilitates the computation of the entanglement of formation in bipartite, entangled GCS. In analogy with the definition of the entangled CS given in (1), the states

$$|\beta,(-1)^{\frac{1}{k}}\beta\rangle=N\left[|\beta\rangle|\beta\rangle+\exp(i\phi)|(-1)^{\frac{1}{k}}\beta\rangle|(-1)^{\frac{1}{k}}\beta\rangle\right], \tag{18}$$

are called the entangled GCS. The normalization constant N is $1/\sqrt{2[1+X^2\cos\phi]}$, where $X^2=2A^{-2}-1$. It is expressible in terms of the two-mode extensions of $|\beta+\rangle$ and $|\beta-\rangle$. The magic basis states to express the entangled GCS are

$$|G_1\rangle=\frac{1}{\sqrt{2}}\left[|\beta+\rangle|\beta+\rangle+|\beta-\rangle|\beta-\rangle\right], \tag{19}$$

$$|G_2\rangle=\frac{i}{\sqrt{2}}\left[|\beta+\rangle|\beta+\rangle-|\beta-\rangle|\beta-\rangle\right], \tag{20}$$

$$|G_3\rangle=\frac{i}{\sqrt{2}}\left[|\beta+\rangle|\beta-\rangle+|\beta-\rangle|\beta+\rangle\right], \tag{21}$$

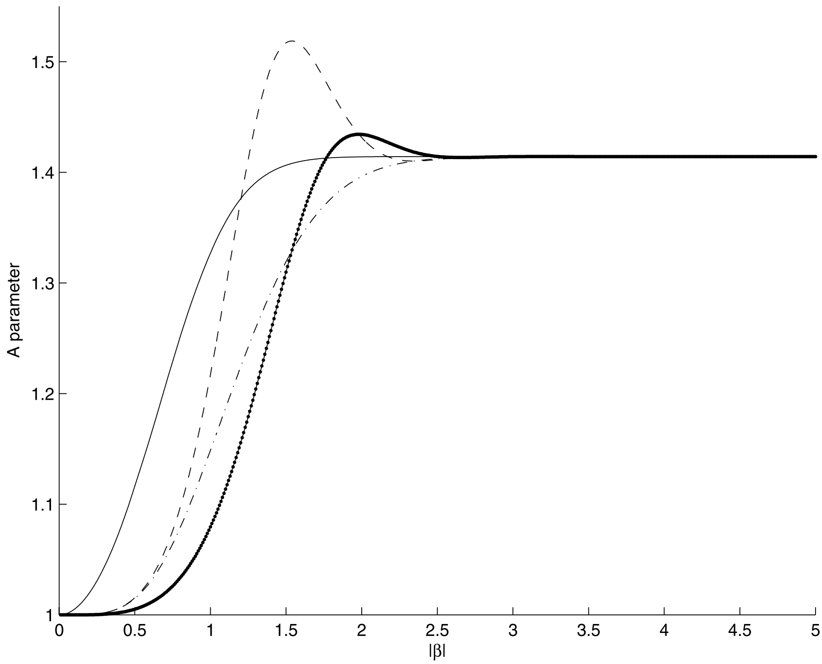

Fig. 1 Parameter A as a function of $|\beta|$. The *thin continuous curve* is for the entangled CS, the *dashed curve* for the entangled ECS, the *dashed-dotted curve* for the entangled squeezed vacuum and the *thick continuous line* for the entangled OCS

$$|G_4\rangle = \frac{1}{\sqrt{2}}\left[|\beta+\rangle|\beta-\rangle - |\beta-\rangle|\beta+\rangle\right]. \tag{22}$$

If the expansion coefficients are $\{\gamma_j\}_{j=1}^4$ in the magic basis $\{|G_j\rangle\}_{j=1}^4$ and the coefficients are $\{a_j\}_{j=1}^4$ in the product basis

$$\{|\beta+\rangle|\beta+\rangle, |\beta-\rangle|\beta-\rangle, |\beta+\rangle|\beta-\rangle, |\beta-\rangle|\beta+\rangle\},$$

then

$$\gamma_1 = \frac{a_1 + a_2}{\sqrt{2}}, \tag{23}$$

$$\gamma_2 = i\,\frac{a_1 - a_2}{\sqrt{2}}, \tag{24}$$

$$\gamma_3 = i\,\frac{a_3 + a_4}{\sqrt{2}}, \tag{25}$$

$$\gamma_4 = \frac{a_3 - a_4}{\sqrt{2}}. \tag{26}$$

Using these relation among the two sets of coefficients, the concurrence C is expressed in terms of the four coefficients a_j as a determinant:

$$C = 2\left|\begin{vmatrix} a_1 & a_3 \\ a_4 & a_2 \end{vmatrix}\right|. \tag{27}$$

The symbol $||\cdot\cdot||$ stands for the absolute value of the determinant. When the relative phase ϕ is zero or π, the coefficients a_j are real for the class of states $|\beta, (-1)^{\frac{1}{k}}\beta\rangle$. In that case the concurrence has the interpretation as the area of a parallelogram with vertices at the origin, (a_1, a_3) and (a_2, a_4).

The entangled GCS is expressed in the basis as follows,

$$\begin{aligned} |\beta, (-1)^{\frac{1}{k}}\beta\rangle = N\Bigg\{&[1+\exp(i\phi)]\left[\frac{1}{A^2}|\beta+\rangle|\beta+\rangle + \frac{1}{B^2}|\beta-\rangle|\beta-\rangle\right] \\ &+\frac{1-\exp(i\phi)}{AB}\big[|\beta+\rangle|\beta-\rangle + |\beta-\rangle|\beta+\rangle\big]\Bigg\}. \end{aligned} \tag{28}$$

The coefficients a_j are easily obtained from this expression. The concurrence between the two modes, computed using (27), is

$$C = \frac{1-X^2}{1+X^2\cos\phi}. \tag{29}$$

It is to be noted that the concurrence depends on the amplitude β since X^2 depends on β. An interesting consequence of the above equation is that all the entangled GCS of the type $|\beta, (-1)^{\frac{1}{k}}\beta\rangle$ with equal value of A possess equal entanglement. For instance, the states with very different physical properties such as an entangled CS and an entangled SV, possess identical entanglement when the respective A parameters are equal. In Fig. 2 the concurrence in the states defined in (18) are shown as a function of the parameter A for different values of ϕ. The fact that the entanglement is one bit for any β when $\phi = \pi$ is obvious from (29); C becomes unity when $\phi = \pi$ and hence the entanglement is one bit.

The derivative of C with respect to X vanishes when $X = 0$, which yields $A = \sqrt{2}$ for all values of ϕ. At this value of A, concurrence is unity and consequently the states $|\beta, (-1)^{\frac{1}{k}}\beta\rangle$ have one bit of entanglement. In essence, the entanglement is one bit for all values of ϕ if $A = \sqrt{2}$. This is complementary to the result that the entanglement is one bit for the GCS for any value of β when $\phi = \pi$.

Different types of entangled GCS with same value of A have same concurrence and hence equal entanglement. However, for a given amplitude β, the value of A differs for the various states, refer Fig. 1. For all the states listed in Table 1, the concurrence between the two-modes becomes unity when $A = \sqrt{2}$. The entanglement as a function of $|\beta|$ is given in Fig. 3. Among the states considered, the two-mode CS attains one ebit of entanglement with smaller value of $|\beta|$ compared to the other states. The two-mode OCS attains one ebit of entanglement at a much larger value of $|\beta|$. However, with $\phi \neq \pi$, the entangled CS and the entangled SV attain one ebit of entanglement when $|\beta|$ becomes infinity. This, in turn, implies that the energy or the mean photon number is infinity. However, the entangled ECS and the entangled OCS attain one bit of entanglement when the respective mean photon numbers are finite. The entangled ECS exhibits a sequence of maxima and minima in the entanglement. These extrema occur when $\tanh(|\beta|^2) = \tan(|\beta|^2)$. The first maximum occurs at $|\beta| \approx 1.26$ and the subsequent minimum at $|\beta| \approx 1.55$. Other extrema are not significantly

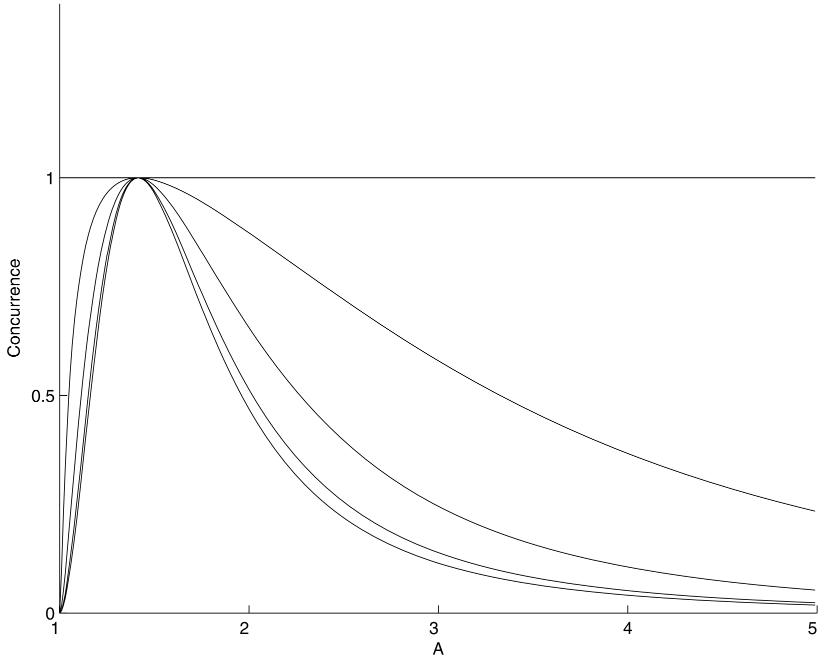

Fig. 2 Concurrence C as a function of the parameter A. The *lowermost curve* corresponds to $\phi = 0$. The *curves above* are for ϕ varied in steps of 0.25π. The *topmost curve* corresponds to $\phi = \pi$ and in this case the concurrence is unity and independent of A. At $A = \sqrt{2}$, concurrence becomes unity for all ϕ

large to be noticed in the figure. From Fig. 1, it is clear that the value of A increases above $\sqrt{2}$ at $|\beta| \approx 1.26$. Consequently, the entanglement becomes one ebit. For the OCS, the first peak in A occurs at $|\beta| \approx 1.9$. For these states, the parameter A exceeds the critical value $\sqrt{2}$ at $|\beta| \approx 1.76$, and the corresponding value of the entanglement is one ebit. Thus, the entangled ECS and the entangled OCS provide examples of GCS attaining one ebit of entanglement for finite values of mean photon number.

The logarithmic states are described by specifying the values of complex parameter γ and the real parameter q whose magnitude is less than unity. The condition for attaining the maximum concurrence is expressed by the following relation between the parameters γ and q:

$$|q| = \sqrt{\exp(|\gamma|^2) - 1}. \tag{30}$$

Since the magnitude of q is limited to unity, the range of values of $|\gamma|$ is restricted to ≈ 0.83255. If γ is larger than this limit, the concurrence does not exhibit a peak. Nevertheless, one bit of entanglement is attained as $|q|$ approaches unity. These features are seen in the curves shown in Fig. 4. The curve that corresponds to $\gamma = 0.1$ exhibits a peak in entanglement at $|q| = 0.1033$, as predicted by the relation in (30). There is no maximum in the curve corresponding to $\gamma = 0.9$ as the corresponding value of $|q|$, satisfying (30), exceeds unity. If $|\gamma| \to \infty$ or $|q| \to 0$, A_{LS} becomes unity making the concurrence and hence the entanglement to vanish. In this limit, the number state expansion of the LS involves only the two-mode vacuum state $|0\rangle|0\rangle$ and the state is separable. It is the case with the other GCS as well due to the fact that both β and $(-1)^{1/k}\beta$ vanish when the amplitude is zero and the resulting superposition involves the state $|m\rangle|m\rangle$ only.

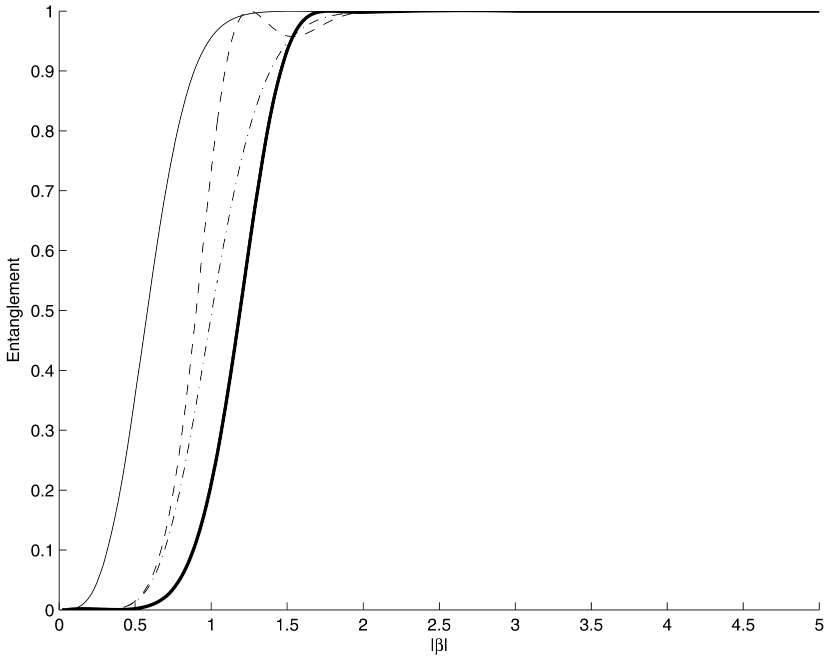

Fig. 3 Entanglement is shown as a function of $|\beta|$. The curves correspond to $\phi = 0.5\pi$. The value of ϕ is chosen so that the dip in the entanglement in the entangled ECS is well pronounced. The *thin continuous curve* is for the entangled coherent state, *dashed curve* for entangled ECS, *dashed-dotted curve* the entangled squeezed vacuum and the *thick continuous line* for the entangled OCS

Yet another type of superposition of the two-mode GCS, linearly independent of the states defined in (18), is

$$N\left[|\beta\rangle|(-1)^{\frac{1}{k}}\beta\rangle + \exp(i\phi)|(-1)^{\frac{1}{k}}\beta\rangle|\beta\rangle\right]. \tag{31}$$

The normalization constant N is $1/\sqrt{2[1+X^2\cos\phi]}$. The relevant a_j coefficients for these states are

$$a_1 = NA^{-2}(1+\exp(i\phi)), \tag{32}$$

$$a_2 = -NB^{-2}(1+\exp(i\phi)), \tag{33}$$

$$a_3 = -NAB(1-\exp(i\phi)), \tag{34}$$

$$a_4 = NAB(1-\exp(i\phi)). \tag{35}$$

The concurrence, computed by substituting the coefficients a_j given above into (27), is the same as for the states specified in (18). Hence, all the properties of entanglement are identical for both the types of states.

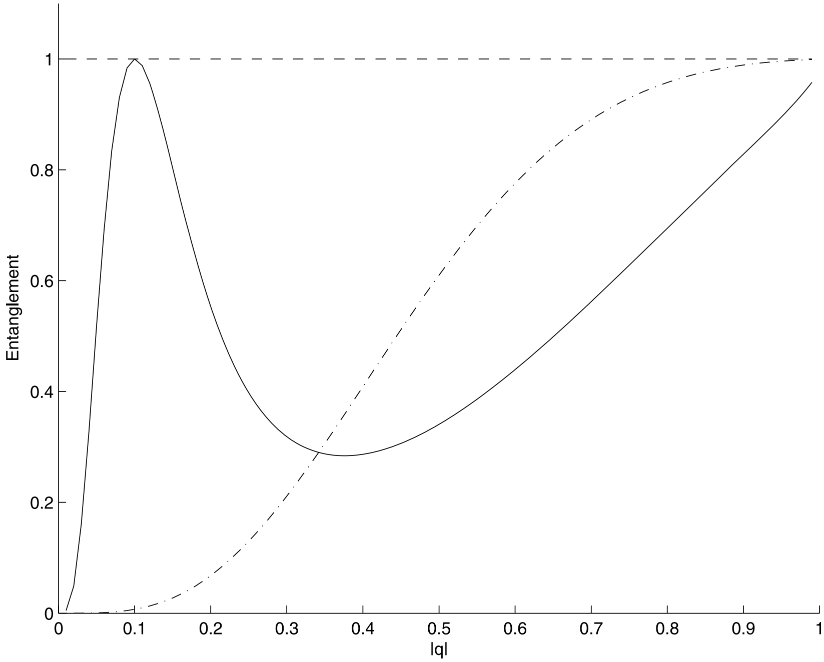

Fig. 4 Entanglement in two-mode logarithmic states is shown as a function of $|q|$. The *continuous curve* corresponds to $\gamma = 0.1$ and the *dotted curve* corresponds to $\gamma = 0.9$. The later case does not exhibit a maximum as the condition in (30) is not satisfied. In both the cases $\phi = 0.5\pi$. The *dashed curve*, which remain constant at unity, corresponds to $\gamma = 0.1$ and $\phi = \pi$

3 Summary

The amount of entanglement in the two-mode GCS is related to the parameter A. All the GCS of equal A possess equal entanglement. When the amplitude of the GCS is such that $A = \sqrt{2}$, the entanglement in the corresponding two-mode case defined in (18) is exactly one bit, irrespective of the relative phase. This general result is complementary to the result that the entanglement is one bit when the relative phase is π. In many cases, one bit of entanglement is attained when the amplitude tends to infinity or the maximum allowed value. There are states, for instance, the entangled ECS, which attain one ebit of entanglement when the amplitude is finite.

References

1. Akhtarshenas, S.J.: Int. J. Theor. Phys. **45**, 2231 (2006)
2. Barnum, H., Knill, E., Ortiz, G., Viola, L.: Phys. Rev. A **68**, 032308 (2003)
3. Bennett, C.H., et al.: Phys. Rev. Lett. **70**, 1895 (1993)
4. Bennett, C.H., et al.: Phys. Rev. A **54**, 3824 (1996)
5. Bennett, C.H., et al.: Phys. Rev. Lett. **76**, 722 (1996)
6. Braunstein, S.L., van Loock, P.: Rev. Mod. Phys. **77**, 513 (2005), and the references therein
7. Buzek, V., Knight, P.L., In: Wolf, E. (ed.) Progress in Optics, vol. XXXIV, p. 1 (1995)
8. Clausen, J., et al.: Phys. Rev. A **66**, 062303 (2002)
9. Clifton, R., Halvorson, H.: Phys. Rev. A **61**, 012108 (1999)

10. Dodonov, V.V., Mankin, I.A., Man'ko, V.I.: Physica **72**, 597 (1974)
11. Eisert, J., et al.: J. Phys. A **35**, 3911 (2002)
12. Heiss, D. (Ed.): Fundamentals of Quantum Information: Quantum Computation, Communication, Decoherence and All That. Lecture Notes in Physics, vol. 587. Springer, Berlin (2002)
13. Hill, S., Wootters, W.K.: Phys. Rev. Lett. **78**, 5022 (1997)
14. Hiroto, O., Sasaki, M.: arXiv:quant-ph/0101018 (2001)
15. Horodecki, M., et al.: Phys. Lett. A **232**, 333 (1997)
16. Horodecki, R., et al.: quant-ph/0702225 (2007), and the references therein
17. Jeong, H., Kim, M.S.: Phys. Rev. A **65**, 042305 (2002)
18. Jeong, H., et al.: Phys. Rev. A **67**, 012106 (2003)
19. Keyl, M., et al.: quant-ph/0212014 (2002)
20. Klauder, J.R., Skagerstam, B.S.: Coherent States: Applications in Physics and Mathematical Physics. World Scientific, Singapore (1985)
21. Knight, P.L., Gerry, G.C.: Introductory Quantum Optics. Cambridge University Press, Cambridge (2005)
22. Lee, J., et al.: New J. Phys. **8**, 23 (2006)
23. Nielsen, M.A., Chuang, I.L.: Quantum Computation and Quantum Information. Cambridge University Press, Cambridge (2002)
24. Paternostro, M.: Phys. Rev. A **67**, 023811 (2003)
25. Perelomov, A.M.: Generalized Coherent States and Their Applications. Springer, Berlin (1986)
26. Peres, A.: Quantum Theory: Concepts and Methods. Kluwer, Dordrecht (1993)
27. Peres, A.: Phys. Rev. Lett. **77**, 1414 (1996)
28. Sanders, B.C.: Phys. Rev. A **45**, 6811 (1992)
29. Scully, M.O., Zubairy, M.S.: Quantum Optics. Cambridge University Press, Cambridge (1997)
30. Simon, R., Satyanarayana, M.V.: J. Mod. Opt. **35**, 719 (1988)
31. Tombesi, P., Mecozzi, A.: J. Opt. Soc. Am. B **4**, 1700 (1986)
32. van Enk, S.J., et al.: Science **279**, 205 (1993)
33. van Enk, S.J., Hirota, O.: Phys. Rev. A **64**, 022313 (2001)
34. van Enk, S.J.: Phys. Rev. Lett. **91**, 017902 (2003)
35. Zhang, W.-M., Feng, D.H., Gilmore, R.G.: Rev. Mod. Phys. **62**, 867 (1990)